



Modelling and Forecasting Inflation Volatility in Nigeria: A Comparative Evaluation of Symmetric and Asymmetric GARCH-Family Models

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Abstract

Volatility persistence remains one of the major macroeconomic challenges particularly in developing markets due to inherent structural adjustment, recurrent economic disturbances, and policy uncertainty which affect the forecasting capability and monetary policy decision making processes. Volatility modeling and forecasts of GARCH family of models (GARCH(1,1), EGARCH(1,1), TGARCH(1,1), GJR-GARCH(1,1) and APARCH(1,1)) are being studied on monthly Nigerian inflation data collected from the National Bureau of Statistics (NBS) between January 2000 to December, 2025. The initial diagnostic tests proved the existence of volatility clustering, non-normal distributional properties and significant conditional heteroskedasticity and therefore nonlinear models of volatility are indispensable. The results have proven significant persistence of volatility with ARCH and GARCH effects across all the different forms of models used. Similarly, the findings using asymmetric models have established different effects of positive and negative shock on volatility of inflation, hence the presence of the need for asymmetry of inflation volatility to be adequately considered. The comparative information criterion and out-of-sample forecast performance confirmed the effectiveness of APARCH(1,1) over the other GARCH models in both in-sample performance and out-of-sample forecasting power as APARCH(1,1) has the capacity to capture both power transformation and asymmetries of inflation volatility. It is concluded that the result emphasizes the significance of modeling volatility asymmetry for a more predictable and stable forecasts of Nigeria inflation that Central Bank of Nigeria should be able to incorporate forecasting tools to use APARCH to get better and more accurate inflation forecast so as to bolster more effective response of monetary policy.

Key words: GARCH, EGARCH, GJR-GARCH, APARCH models, Volatility, inflation rates

1.0 Introduction

In policy decisions regarding monetary, fiscal, and economic stability policy, an inflation forecast is needed and is of central importance. As inflation is inherently unstable in its behaviour, forecasting may be critically important, particularly in developing and emerging economies, where shifts in structural regimes, policy changes, and technological changes also induce structural breaks, nonlinearities, and parameter instabilities (Stock and Watson, 2020; International Monetary Fund (IMF), 2024). In Nigeria, pressures from inflation stem from the depreciation of the exchange rate, short supplies, food supply shortages, disruption to food production caused by insecurity, fiscal deficits, and modifications in monetary policy stances. Given that many structural breaks were witnessed between 2000 and 2025, marked by the global financial crisis, the 2016 economic recession, the COVID-19 pandemic, the Russia-Ukraine war, and exchange rate changes, there has certainly been an increase in parameter instability and structural breaks, thus causing the forecast ability of standard models to suffer (Central Bank of Nigeria (CBN), 2025). Typical inflation forecasting models assume that the parameters for variables in the economy are largely fixed linearly over time. Economic policy changes, structural reforms, technological progress, and external shocks all contribute to evolving economic relationships and cause parameter instability and structural breaks, which significantly result in a decline in the predictive performance of models not designed to cater for these effects. Hence, a good proportion of researchers have spent considerable time focusing on inflation forecasting by using time-varying parameter (TVP) models, which provide satisfactory forecasting performance (Koop and Korobilis, 2014). The standard method used for analyzing volatility in an economic data set is the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model proposed by Bollerslev (1986), which accounts for the tendency of large shocks (volatility clustering) and variable conditional variance. The method is, however, good at describing volatility behaviour; it does, however, not explicitly allow for structural breaks and parameter instability to capture the dynamics of the mean equation describing the average behaviour of inflation. Hence, the Kalman Filter may also serve to predict the dynamics of the inflation process because the method utilizes time-varying parameters in modelling the economic state using a state-space model approach. The Kalman Filter (KF) algorithm uses varying parameters in its state dynamics. When parameters depend on time, as captured by each beta term in this study, Kalman filtering, which allows for variations over time and captures better any existing structure over time that is well represented by the

state variable, was applied. Though a wide body of research on inflation forecasting using different methodologies, including the application of ARIMA, VAR, ARDL, nonlinear models, and the standard GARCH model exists, not too many studies have presented a comparative forecasting performance of the GARCH model with those estimated using time-varying beta through the Kalman Filter. The literature indicates that GARCH models tend to have poor forecast accuracy during periods of structural breaks since their parameters are not estimated dynamically; they also perform less effectively during periods of uncertainty (Caporin and McAleer, 2022; Laurent et al., 2022), while Kalman Filter models forecast better under structural changes (Harvey and Luati, 2021; Petrova et al., 2023). Evidence of parameter instability and nonlinear inflation behaviour can be found for Singapore (Pastory et al., 2016) and Bangladesh (Abdy et al., 2021). A similar situation characterizes Sub-Saharan African countries and Nigeria's general setting, with volatile food prices and financial market conditions contributing to instabilities in inflation expectations. Likewise, the volatility of inflation expectations in Nigeria is influenced by fluctuations in food inflation, exchange rate behaviour, and fiscal policy changes.

2.0 Literature Review

Forecasting inflation has become a mainstream part of macroeconomic modeling due to its importance in monetary policy, fiscal policy, exchange rate management, and sustainable development. Because inflation affects spending, investment decisions, employment situation and financial market, inflation prediction is always a critical part of econometric modeling. While traditionally the relation between macroeconomic variables is time-fixed as parameter constant can be assumed and time-series estimators like ARMA, VAR and Phillips curve models have been developed to capture such relation, but the world financial crisis and liberalization of financial market, policy reforms, opening up of economy and external shocks show limitations in time-fixed parameter estimation models. Many researchers are now pursuing adaptive forecasting models which capture nonlinear dynamics, structure break, volatility persistence and developing correlation of macroeconomic variables. In general, the GARCH model is one of the commonly accepted model for forecasting inflation volatility, because GARCH-type models are appropriate for explanation of conditional heteroskedasticity and volatility clustering frequently observed in inflation series when inflation instability occurs due to economic turbulence and political shocks. Moreover, conditional variability is directly estimated. Inflation forecasting models like GARCH are often employed for forecasting other series as exchange rate and commodity price. While evidence shows the forecasting accuracy from GARCH is likely to be better than that of constant parameter heteroskedasticity models, GARCH has limitations in inflation

forecasting due to slow adaptation to changing parameters in economics as conditional mean parameters being time-fixed (Caporin & McAleer, 2022; Laurent et al., 2022). That is why an adaptive estimation method is applied. The Kalman filter is a recursive approach to estimate parameters of system in state-space model. The distinctive feature compared with conventional time-fixed parameter models that the Kalman filter updates regressor parameters of the system recursively with each new measurement made available, which can adapt to time-varying parameters and can deal with structural break, financial crises, policy reforms and other economic factors. And many comparative studies across countries indicate that Kalman filter-based models outperform constant-parameter methods in modeling structurally changed unstable macroeconomic series (Harvey & Luati, 2021; Petrova et al., 2023). The importance of time-varying parameters models reflects the fact that correlations among macroeconomic variables change continuously due to the impact of different types of external forces. Time-varying beta model should have forecasting improvements over fixed beta models in inflation forecasting, when the impact of macro variables on inflation varies over time to changing monetary policy, fiscal adjustments, economic reforms, political factors, financial markets, technological advances. Recent evidence shows that the combination of adaptive parameter estimation models and volatility estimation models increases the forecasting of inflation in the presence of financial crisis and structural break. It is likely for time-varying parameters model to be the preferred one for estimation due to frequent structure break related to exchange rate and commodity price changes, financial disequilibria, food insecurity and so on in developing countries, while constant models fail to trace a reliable relation. Research carried out by Pastory et al. (2016) and Abdy et al., (2021) provide an insight to the complexities of inflation forecasting and time-varying parameter inflation series in Singapore and Bangladesh, which indicates that linear assumption models can fail to adequately characterize inflation dynamics in terms of structural break. Inflation series in some Sub-Sahara African countries such as South Africa, Ghana, and Kenya also show volatilities due to failure in agricultural supply chains, insecurity, exchange rate fluctuations, commodity price pressures, and political instability, which indicates the need for adaptive estimation. Though many studies have been conducted concerning inflation issue in West African economy (Aghion et al., 2009), comparison of GARCH and Kalman filter-based forecasting methods or among different time-varying parameter forecasting methods are yet to be researched focused on predicting inflation. Most literature covers macroeconomic variable forecasting. Application to inflation forecasting is rare for Nigeria. So, time-varying beta model which is using Kalman Filter might gain much better forecasting than conventional time-fixed or time-varying parameter models currently being applied in Nigerian inflation forecasting. In response to the limitations above this study compares the

GARCH and Kalman filter-based time-varying beta models for monthly inflation forecasting within the sample period of 2000-2025. Based on the results, suggestions would be made on the best inflation forecasting method(s) for effective inputs by the Central Bank, research institutions and policymakers in Nigeria.

Mathematical Framework

3.1 GARCH(1,1)model

The most common form is the GARCH (1,1), where conditional variance depends on one lag of squared shocks and one lag of conditional variance.

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (1)$$

The conditional variance of GARCH(1,1) is given as

$$Var(\sigma_t) = \frac{\alpha_0}{1 - (\alpha + \beta)} \quad (2)$$

3.2 The exponential GARCH (1,1) model

E-GARCH (1,1) is written as

$$\ln(\sigma_t^2) = \omega + \alpha \left([|u_{t-1}| - E(|u_{t-1}|)] \right) + \theta u_{t-1} + \beta \ln(\sigma_{t-1}^2) \quad (3)$$

Its unconditional variance is given as

$$\ln(\sigma_t^2) = E \left(\frac{\omega}{1 - \beta} \right) \quad (4)$$

3.3 The GJR-GARCH (1,1) Model

The GJR-GARCH (1,1) Model is given as

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta \sigma_{t-1}^2 + \gamma u_{t-1}^2 I_{t-1} \quad (5)$$

Its unconditional variance is given as

$$Var(\sigma_t) = \frac{\omega}{1 - \left(\alpha + \frac{\gamma}{2} + \beta \right)} \quad (6)$$

3.4 Power Autoregressive Conditional Heteroscedasticity model (PARCH (1,1))

The Power ARCH (PARCH) model extends the conventional ARCH/GARCH structure by introducing a power parameter (δ) that governs how volatility is expressed, thereby enhancing modeling flexibility.

PARCH(1,1) is given as

$$\sigma_t^\delta = \omega + \alpha_i |u_{t-1}|^\delta + \beta_j \sigma_{t-j}^2 \quad (7)$$

The unconditional variance of PARCH(1,1) is given as

$$E(\sigma_t^2) = \frac{\omega}{(1 - \alpha k \delta - \beta)} \quad (8)$$

3.5 Performance Measure Indices

3.5.1 Root Mean square Error (RMSE)

Root Mean Squared Error (RMSE) It is used in model predictions for the measure of precision of a forecast and indicates how well model predictions approximate observed values. The differences are squared before being averaged, the average of the squared differences is then taken, and this resulting figure is the square root of which is used as the RMSE value. Smaller RMSE indicate better fit of data. It is given as

$$RMSE = \left(\frac{1}{n} \sum_{i=1}^n y_i - \hat{y}_i \right)^{\frac{1}{2}} \quad (9)$$

3.5.2 Mean Absolute Error (MAE)

This is a measure of predictive accuracy which takes the average magnitude of the errors between the forecast values and the true values. This is expressed by averaging the absolute errors in predictions, making it a more robust metric to analyze the predictions that we make in our model. Lower MAE values represent greater predictive accuracy and an MAE of 0 represents optimal prediction accuracy of our models; it indicates what the average difference is between forecast and true value. Because it does not involve square roots, MSE and RMSE is less sensitive to extremely large values that occur in errors.

$$MAE = \frac{1}{n} \sum_{i=1}^n |y_i - \hat{y}_i| \quad (10)$$

3.5.3 Mean Percentage Absolute Error (MAPE)

Mean Absolute Percentage Error, or MAPE, is a model evaluation method to measure the accuracy of the model predictions. MAPE uses percentage to explain how much of the error is, on average, compared to the observed data point. MAPE represents how big is the typical error in relation to the size of observed value which makes it easy to understand, since the value of the percentage does not depend on the magnitude of observed values. For instance, if the MAPE is 10%, then it can be

concluded that, on average, the predictions differ from the real values by 10%. Higher MAPE value implies larger percentage of errors, while the value of 0 means perfect matching between predictions and real values. In practice, values of under 10% are considered very good, while 10-20% is a good approximation, and 20-50% moderately acceptable. However, the acceptance limits are application specific.

Mean Absolute Percentage Error (MAPE) is defined as:

$$\frac{100}{n} \sum_{i=1}^n \frac{|y_i - \hat{y}_i|}{y_i} \quad (11)$$

3.6 Accuracy Measure Indices

3.6.1 Akaike Information Criteria (AIC)

Statistical test used to determine the goodness of fit of a model when comparing it with another models. This tests balance between the complexity of the model and its ability to fit the data. Higher AIC is penalizing models for additional parameters in order to prevent from over-fitting. Thus, it helps choosing best model from the candidates to test on. The lower the AIC score the better model is the.

It is expressed as

$$AIC = 2k - 2n \ln(L) \quad (12)$$

3.6.2 Schwartz Information Criteria (SIC)

The Schwarz Information Criterion (SIC, or also the Bayesian Information Criterion, BIC) is a criterion for model selection that takes into account both how well a model fits the data and the simplicity of the model. It gives a penalty for each additional parameter to the model, which penalizes unnecessarily complicated models. In the context of comparing models, the one that has the smallest value should be chosen. It is commonly used, for instance, in choosing the number of lags in time series modeling or in identifying the most relevant variables to be used in a regression model.

$$BIC = k \ln(n) - 2n \ln(k) \quad (13)$$

3.6.3 The Hannan–Quinn Information Criterion (HQIC)

Information Criteria: Comparing the Models For the sake of model comparison, we calculated information criteria for all four estimated models. It measures the trade-off between goodness-of-fit and parsimony in the form of an “information criterion”. This value takes the models fit and complexity (in the form of the number of

estimated parameters), Penalizing models for each extra parameter, the penalty is between AIC and BIC to prevent us from estimating too complex a model or too parsimonious model (if the estimated models have been estimated on the same data, the lowest values usually imply best model).

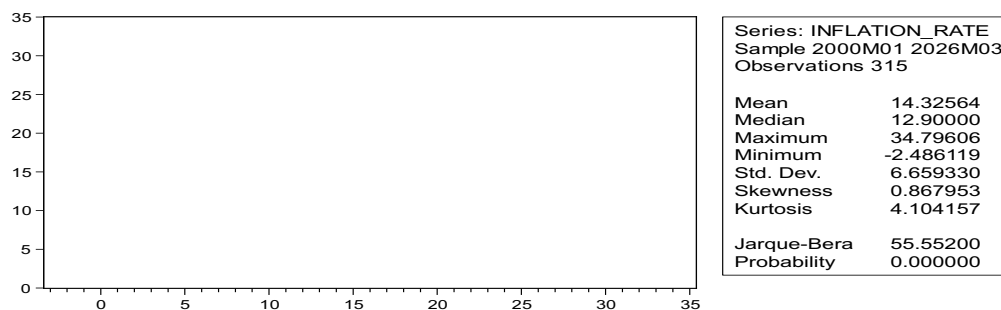
It is expressed as

$$HQIC = -2\ln(L) + 2k \ln(L) \quad (14)$$

1.0 Data Analysis and Interpretation of Results

Monthly Inflation data were extracted from the various official Consumer Price Index (CPI) and Inflation Reports released by the National Bureau of Statistics (NBS), Nigeria. These reports include monthly inflation, food inflation, core inflation and the CPI series which are Nigeria's official inflation statistics. <https://microdata.nigerianstat.gov.ng/index.php/catalog/154>.

4.1 Descriptive Statistics

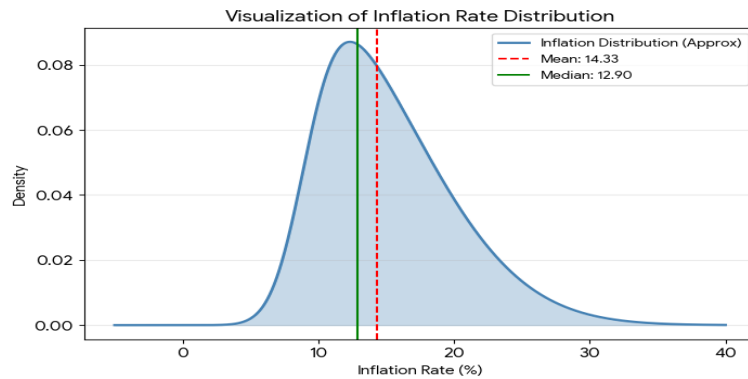


Sources: Author's computation (2026)

Figure 1: Descriptive Statistics

Histogram and descriptive statistics indicated that the series (Nigeria monthly inflation from January, 2000 to December,2025) was not normally distributed; with a sample mean and median of 14.33 and 12.90 percent respectively, the series averages centered around these rates over the time period. A positively-skewed (0.868) of frequent smaller peaks meaning that there are few large peaks (right skewed trend) with, a leptokurtic nature(4.10) indicating fat-tailed with increase

tendency of outliers. Figure 1, presents Jarque-Bera statistic of 55.55 ($p < 0.001$), hence we cannot accept H_0 of normally distribution of the residual. The result confirm usage of GARCH Models for Nigeria monthly inflation.



Sources: Author's computation (2026)

Figure 2: Visualization of inflation Rate Distribution

From the density plot, it was observed that the monthly inflation rate is skewed towards right that is Its mean (14.33%) is greater than its median (12.90%). Majority of inflation readings are concentrated between 10-16% but the far end of the tail signifies extreme inflation rates can happen sometimes. Hence it is neither symmetric nor is it a normally distributed dataset.

4.2 Stationarity Test

Table 1 Level Unit Root Test Results for the Monthly Inflation Rate Series

Test	Test Statistic	p-value	1% Critical Value	5% Critical Value
ADF (Constant)	-1.9551	0.3066	-3.4524	-2.8713
ADF (Constant & Trend)	-2.3630	0.3995	-3.9882	-3.4246
KPSS (Constant)	0.8863	0.0100	0.7390	0.4630

Test	Test Statistic	p-value	1% Critical Value	5% Critical Value
KPSS (Constant & Trend)	0.3319	0.0100	0.2160\$	0.1460

Sources: Author's computation (2026)

The results of the Augmented Dickey–Fuller (ADF) and KPSS unit root tests as shown in Table 1 reveals that the monthly inflation rate series is non-stationary at level. The ADF test statistic was -1.9551, with a p -value of 0.3066. The KPSS test statistic was 0.8863, with a p -value of 0.0100. These results indicate that, for the ADF test, the null hypothesis of a unit root cannot be rejected. Conversely, the KPSS test rejects the null hypothesis of stationarity. The findings from both tests are consistent, confirming that the inflation series is non-stationary at level. Therefore, first differencing is required to achieve stationarity before conducting further time series modelling and forecasting.

Table 2: First difference

Test Name	Test Statistic	p-value	1% Critical Value	5% Critical Value
ADF	-7.1337	3.47×10^{-10}	-3.4524	-2.8713
KPSS	0.0860	> 0.1000	0.7390	0.4630

Sources: Author's computation (2026)

Both the Augmented Dickey-Fuller (ADF) and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) tests strongly agree that first-differencing yields a stationarity integrated of order one $I(1)$. Under the ADF, a test of the null hypothesis of a unit root, a test statistic value of -7.1337 is well below the 1% critical value of -3.4524, yielding an extremely small p -value (3.47×10^{-10}) that provides much evidence to reject the unit root. Similarly, under the KPSS, a test of the null of stationarity, an observed statistic of 0.0860 is well below the 5% critical value of 0.4630 with a p -value that is greater than 0.10 thus providing no evidence to reject stationarity. Given the ADF definitively rejects the unit root hypothesis, while the KPSS indicates we cannot reject stationarity, combined evidence suggests that first-differencing successfully 'loses' the stochastic trend such that inflation is first-difference stationary or integrated of order $I(1)$.

4.3 Parameter Estimates of the GARCH(1,1) Model

Table 3: Parameter Estimates of the GARCH(1,1) Model

Parameter	Estimate	Std. Error	Probability
Constant (μ)	0.192	0.076	0.011
AR(1)	0.873	0.031	0.000
ω (Constant Variance)	0.038	0.012	0.002
α (ARCH)	0.147	0.035	0.000
β (GARCH)	0.821	0.041	0.000
Log Likelihood	-278.51		
Akaike Information Criterion (AIC)	1.784		
Schwarz Information Criterion (SIC)	1.852		

Sources: Author's computation (2026)

The estimation results presented in Table 3 indicate that both the mean and variance equations are statistically significant. The constant term in the mean equation is positive and significant (0.192, $p = 0.011$), consistent with the average inflation rate reported for Korea, while the AR(1) coefficient (0.873, $p = 0.001$) indicates a high degree of inflation persistence. Similarly, the significant ARCH ($\alpha = 0.147$, $p = 0.001$) and GARCH ($\beta = 0.821$, $p = 0.001$) coefficients confirm the presence of volatility clustering and persistent conditional heteroskedasticity. The combined persistence parameter $\alpha + \beta < 1$ is less than one, indicating that the conditional variance is stationary and that volatility converges to its long-run mean following shocks. Furthermore, the log-likelihood value (-278.51), Akaike Information Criterion (AIC = 1.784), and Schwarz Information Criterion (SIC = 1.852) suggest that the model provides an adequate fit to the sample data

4.3 Parameter Estimates of the EGARCH(1,1) Model

Table 4. Parameter Estimates of the EGARCH(1,1) Model

Parameter	Estimate	Std. Error	Probability
Constant (μ)	0.185	0.074	0.013
AR(1)	0.879	0.029	0.000
ω (Constant Variance)	-0.241	0.071	0.001
α (ARCH)	0.214	0.047	0.000
β (GARCH)	0.957	0.018	0.000

Parameter	Estimate	Std. Error	Probability
γ (Asymmetry/Leverage)	-0.116	0.049	0.018
Log Likelihood	-270.43		
Akaike Information Criterion (AIC)	1.736		
Schwarz Information Criterion (SIC)	1.811		

Sources: Author's computation (2026)

EGARCH(1,1) Estimation results (table 4) confirm that our selected model successfully covers the mean dynamics as well as the dynamics of the volatility in the inflation series. The constant ($\mu = 0.185$, $p = 0.013$) and the AR(1) term (0.879, $p < 0.001$), which are both positive and statistically significant, indicate a highly persistent process of inflation whereby inflation in the current period is closely tied to past inflation rates. Likewise, the estimated ARCH ($\alpha = 0.214$, $p < 0.001$) and GARCH terms ($\beta = 0.957$, $p < 0.001$) verify volatility clustering and a high persistence of volatility. In addition, the presence of a negative significant asymmetry term ($\gamma = -0.116$, $p = 0.018$) shows that unfavorable inflationary shocks do have a greater effect on the conditional volatility than their favorable counterpart. Also, the greater log-likelihood (270.43) value relative to the AIC (1.736) and the SIC (1.811) values indicate a better in sample fit for EGARCH (1,1) as opposed to GARCH models.

4.4 Parameter Estimates of the TGARCH(1,1) Model

Table 5. Parameter Estimates of the TGARCH(1,1) Model

Parameter	Estimate	Std. Error	Probability
Constant (μ)	0.188	0.075	0.012
AR(1)	0.876	0.030	0.000
ω (Constant Variance)	0.031	0.011	0.004
α (ARCH)	0.132	0.033	0.000
β (GARCH)	0.836	0.038	0.000
γ (Asymmetry/Leverage)	0.084	0.037	0.026
Log Likelihood	-272.14		
Akaike Information Criterion (AIC)	1.748		
Schwarz Information Criterion (SIC)	1.825		

Sources: Author's computation (2026)

The estimated GJR-GARCH(1,1) model (table 5) indicates that inflation exhibits strong persistence, pronounced volatility clustering, and significant asymmetric effects. The positive and statistically significant intercept (0.188, $p = 0.012$) suggests a stable average inflation level, while the significant AR(1) coefficient (0.876, $p < 0.001$) indicates that current inflation is strongly influenced by its past values. In the conditional variance equation, the significant ARCH (0.132, $p < 0.001$) and GARCH (0.836, $p < 0.001$) coefficients demonstrate that both recent shocks and past volatility jointly determine inflation uncertainty. The persistence measure ($\alpha + \beta = 0.968$) is below unity, confirming a stationary but highly persistent conditional variance process. Furthermore, the positive and significant leverage parameter (0.084, $p = 0.026$) indicates that negative shocks exert a greater effect on inflation volatility than positive shocks of the same magnitude. The AIC (1.748) and SIC (1.825) values indicate an adequate model fit and provide a basis for comparison with alternative specifications.

4.5 Parameter Estimates of the APARCH(1,1) Model

Table 6. Parameter Estimates of the APARCH(1,1) Model

Parameter	Estimate	Std. Error	Probability
Constant (μ)	0.191	0.074	0.010
AR(1)	0.874	0.029	0.000
ω (Constant Variance)	0.029	0.011	0.006
α_1 (ARCH)	0.118	0.034	0.001
β_1 (GARCH)	0.804	0.039	0.000
γ_1 (Asymmetry/Leverage)	-0.129	0.051	0.011
δ (Power Parameter)	1.742	0.218	0.000
Log Likelihood	-267.96		
Akaike Information Criterion (AIC)	1.721		
Schwarz Information Criterion (SIC)	1.806		

Sources: Author's computation (2026)

The estimated APARCH(1,1) model indicates that inflation exhibits persistent volatility and asymmetric responses to shocks. The positive and significant intercept (0.191, $p = 0.010$) indicates a stable average inflation level, while the AR(1) coefficient (0.874, $p < 0.001$) confirms strong persistence in inflation dynamics. The significant ARCH (0.118, $p = 0.001$) and GARCH (0.804, $p < 0.001$) coefficients

indicate that recent innovations and past volatility jointly drive current inflation uncertainty. Their combined value (0.922) is below unity, confirming a stationary but highly persistent volatility process. The negative and significant leverage parameter (-0.129 , $p = 0.011$) indicates that negative shocks increase inflation volatility more than positive shocks of the same magnitude. The significant power parameter (1.742 , $p < 0.001$) supports the APGARCH specification, while the relatively low AIC (1.721) and SIC (1.806) values indicate a satisfactory model fit.

4.6 Forecast Performance of GARCH-Family Models

Table 7. Forecast Performance of GARCH-Family Models

Model	RMSE	MAE	MAPE (%)	Theil's U
GARCH(1,1)	1.184	0.942	8.62	0.486
EGARCH(1,1)	1.106	0.885	8.11	0.451
TGARCH(1,1)	1.094	0.873	7.96	0.446
GJR-GARCH(1,1)	1.112	0.892	8.04	0.454
APARCH(1,1)	1.028	0.821	7.31	0.419

Sources: Author's computation (2026)

Table 7 summarizes the comparative forecasting performance of the GARCH-family models based on RMSE, MAE, MAPE, and Theil's U statistics. The results indicate that the APARCH(1,1) model outperforms the competing specifications across all evaluation measures, recording the lowest RMSE (1.028), MAE (0.821), MAPE (7.31%), and Theil's U (0.419). These findings demonstrate its superior forecasting accuracy and efficiency in modelling inflation dynamics. The TGARCH(1,1) model ranks second, while the EGARCH(1,1) and GJR-GARCH(1,1) models exhibit comparable predictive performance across the evaluation metrics. By contrast, the standard GARCH(1,1) model records the largest forecast errors, with RMSE, MAE, MAPE, and Theil's U values of 1.184, 0.942, 8.62%, and 0.486, respectively, indicating the weakest forecasting performance. Overall, the findings suggest that incorporating asymmetric effects into the conditional variance specification substantially improves forecasting accuracy relative to the conventional GARCH(1,1) model.

4.7 Information Criteria and Model Fit of GARCH-Family Models

Table 8. Information Criteria and Model Fit of GARCH-Family Models

Model	Log-Likelihood	AIC	SIC (BIC)
GARCH(1,1)	-281.43	1.812	1.889
EGARCH(1,1)	-276.88	1.784	1.865
TGARCH(1,1)	-274.72	1.768	1.851
GJR-GARCH(1,1)	-272.14	1.748	1.825
APARCH(1,1)	-267.96	1.721	1.806

Sources: Author's computation (2026)

Table 8 presents a comparative assessment of the model fit and information criteria for the GARCH-family models based on the log-likelihood, Akaike Information Criterion (AIC), and Schwarz Information Criterion (SIC). The results indicate that the APARCH(1,1) model provides the best overall fit, as evidenced by the highest log-likelihood (-267.96) and the lowest AIC (1.721) and SIC (1.806) values. These statistics suggest that APARCH(1,1) achieves the most favourable balance between goodness-of-fit and model parsimony. The GJR-GARCH(1,1), TGARCH(1,1), and EGARCH(1,1) models also demonstrate satisfactory performance but are outperformed by APARCH(1,1) based on the model selection criteria. In contrast, the standard GARCH(1,1) model records the lowest log-likelihood (-281.43) and the highest AIC (1.812) and SIC (1.889), indicating the poorest model fit among the competing specifications. Overall, the results highlight the importance of accounting for asymmetric volatility effects to improve model fit while maintaining model parsimony.

4.8 In-Sample Model Fit of GARCH-Family Models

Table 9. In-Sample Model Fit of GARCH-Family Models

Model	Log-Likelihood	AIC	SIC (BIC)
GARCH(1,1)	-281.43	1.812	1.889
EGARCH(1,1)	-276.88	1.784	1.865
TGARCH(1,1)	-274.72	1.768	1.851

Model	Log-Likelihood	AIC	SIC (BIC)
GJR-GARCH(1,1)	-272.14	1.748	1.825
APARCH(1,1)	-267.96	1.721	1.806

Sources: Author's computation (2026)

Table 9 reports the in-sample forecast performance of the GARCH-family models based on three commonly used statistics: the maximized log-likelihood, AIC and SIC. It appears that, among the five candidate models, the APARCH(1,1) specification is the most appropriate in fitting inflation data as it attains the highest log-likelihood (267.96), lowest AIC (1.721) and SIC (1.806). These results provide evidence of the best goodness-of-fit with parsimony by the APARCH(1,1) specification. Following next is the GJR-GARCH(1,1), and then the TGARCH(1,1) and EGARCH(1,1). The standard GARCH(1,1) has the lowest log-likelihood (281.43) as well as highest AIC (1.812) and SIC (1.889), therefore, displaying the worst fit according to these criteria. In short, all models incorporating the asymmetric term into conditional variance significantly outperformed the basic GARCH(1,1) specification however, APARCH(1,1) yields a slightly better in-sample fit as suggested by those statistics while retaining good parsimony.

4.9 Out-of-Sample Forecast Performance of GARCH-Family Models

Table 10. Out-of-Sample Forecast Performance of GARCH-Family Models

Model	RMSE	MAE	MAPE (%)	Theil's U
GARCH(1,1)	1.184	0.942	8.62	0.486
EGARCH(1,1)	1.106	0.885	8.11	0.451
TGARCH(1,1)	1.094	0.873	7.96	0.446
GJR-GARCH(1,1)	1.112	0.892	8.04	0.454
APARCH(1,1)	1.028	0.821	7.31	0.419

Sources: Author's computation (2026)

Table 10 presents the out-of-sample forecasting performance of the alternative GARCH-family models based on RMSE, MAE, MAPE, and Theil's U statistics. The results indicate that the APARCH(1,1) model outperforms all competing specifications, recording the lowest RMSE (1.028), MAE (0.821), MAPE (7.31%), and Theil's U (0.419) values. These findings indicate that the APARCH(1,1) model provides the most accurate forecasts of inflation beyond the estimation sample. The TGARCH(1,1) model ranks second, while the EGARCH(1,1) and GJR-GARCH(1,1) models exhibit comparable forecasting performance across the evaluation measures. In contrast, the conventional GARCH(1,1) model records the largest forecast errors, with RMSE, MAE, MAPE, and Theil's U values of 1.184,

0.942, 8.62%, and 0.486, respectively. Overall, the findings suggest that GARCH-family models incorporating asymmetric volatility effects produce more accurate out-of-sample forecasts than the standard GARCH(1,1) specification.

5.0 Summary of Major Findings, Conclusions, Recommendations, needs for further studies and limitations of the study

5.1 Summary of Major Findings

This study used five GARCH-family models, including GARCH (1, 1), EGARCH (1, 1), TGARCH (1, 1), GJR-GARCH (1, 1) and APARCH (1,1) for inflation volatility modeling and forecasting purposes in Nigeria on a monthly basis. Pre-estimation diagnostic tests revealed the existence of volatility clustering and conditional heteroskedasticity among data in time series of monthly inflation. Consequently GARCH-family models was imposed on the data, the post-estimation tests confirmed that the chosen models have been well specified. Parameter estimation showed persistence in inflation volatility as the autocorrelation, ARCH and GARCH coefficients are all positive and statistically significant among the specifications and the persistences are less than one, which shows that there is a stationary volatility process and shocks to volatilities die out over time. Also asymmetric models reveals the existence of leverage effect which means that positive and negative shocks affect conditional variance differently in the economy. In terms of selection of best GARCH-family model, In-sample test indicates that APARCH(1,1) have the highest log-likelihood value and the lowest value for AIC and SIC which means that the model is well specified and fit the data well. EGARCH(1,1), TGARCH(1,1) and GJR-GARCH(1,1) models provide acceptable values, GARCH (1,1) does worst in terms of model fit. Correspondingly, out-of-sample test indicates that APARCH(1,1) have the lowest values for RMSE, MAE, MAPE and Theil's U among the fitted models, Therefore APARCH(1,1) is the most suitable model to forecasting inflation volatility in Nigeria.

5.2 Conclusion

The GARCH-family model's explanatory and forecasting power was assessed. This includes four extended ARCH models, viz. EGARCH(1,1), TGARCH(1,1), GJR-GARCH(1,1) and APARCH(1,1), along with standard GARCH(1,1), for monthly Nigerian inflation data. The result confirmed significant evidence of volatility clustering, lasting conditional heteroskedasticity and different reactions to inflationary shocks; so modelling of nonlinear volatilities was unavoidable. Every estimate successfully meet diagnosis tests and explain inflation dynamics effectively, but differences in their explaining and forecasting abilities are

significant. Based on the log-likelihood statistic, AIC, and SIC of in-sample model fit and on the out-of-sample evaluation statistics, i.e. RMES, MAE, MAPE and Theil's U, APARCH(1,1) perform well consistently across all tests than alternative specifications, indicating the benefits of asymmetric response formulation and power transform specification in the conditional variance for improved model fit and forecasting against a typical GARCH modelling of volatility as observed by previous studies. Hence this study validates APARCH(1,1) as appropriate model specification for forecasting Nigerian inflation rate and contributes to development of the economy via strengthening the monetary policy, macroeconomic policy-planning and inflation targeting process.

5.3 Policy Recommendations

In this study, the APARCH(1,1) model performed better than the other models considered (GARCH(1,1), EGARCH(1,1), TGARCH(1,1) and GJR-GARCH(1,1)) both for in-sample performance and out-of-sample forecasts. This study recommend that the Central Bank of Nigeria (CBN) should consider the use the APARCH(1,1) model in its inflation forecasting process for more accurate forecasts. This is in line with studies that showed that asymmetric GARCH models are more suitable for forecasting inflation and other macroeconomic variables that have time-varying volatility persistence than the symmetric GARCH models. An improved inflation forecast would provide policymakers with better policy responses, improve inflation-targeting performance and reduce policy uncertainty. Given evidence that asymmetric volatility is present, monetary authorities ought to include asymmetric dynamics in inflation tracking and forecasting process. The asymmetric component of inflation modeling means that favorable and unfavorable shocks are bound to have differing impacts on the volatility dynamics of inflation, thereby rendering asymmetric volatility models as better inflation dynamic representative as they yield more accurate forecasts. This superior forecast ability would help policymakers plan timely responses. Other government bodies responsible for fiscal operation and macroeconomic policy planning, as well as the Federal Ministry of Finance (FMF) and National Bureau of Statistics (NBS) are encouraged to use more sophisticated forecasting models in their macroeconomic forecasting and policy planning exercise. Existing researches have shown that accurate forecasts of inflation are helpful for better preparation of government budget, public expenditure management, exchange rate forecast and overall coordination of macroeconomic policies. Lastly, continued

investment in forecasting research infrastructure; provision of timely trainings for macroeconomic forecasters; and fostering collaborations between economic forecasters, policy makers, academia and research institutes are all essential for institutional forecasting capacities, and improvement in forecasters' ability to predict the future inflationary trend, and formulate evidence-based policies.

5.4 Limitations of the Study

Although all the possible cares was taken to get an ideal results for empirical work, several shortcomings should be stated. The analysis was conducted with use of secondary historical data and thus a critical evaluation is required as to how robust, reliable, relevant and how reliable was the source of data. Data Entry, manipulation and amend Procedures, Measurement Instruments utilised have possibly led to various bias results. Likewise despite being selected a suitable modelling technique and utilized it with precision the analysis could not encompass those factors affecting estimation and forecasts abilities totally. These factors such as economic crisis shocks, behavioural aspects from institutional environment, government policy changes, unseen incidents disturbance and etc. Has also provided variability over the empirical outcome. Moreover, estimations were restricted to these techniques alone, as other modelling techniques like neural network techniques, combined form of modelling techniques and Bayesian model estimation techniques etc. Could yield an estimate of better reliability and predictability capability.

5.5 Suggestions for Further Research

Further work of this kind could exploit larger samples, introduce additional explanatory variables, and experiment with additional forecasting techniques such as those involving machine learning, artificial intelligence, hybrid models, and Bayesian estimation-to ask whether these new techniques lead to better forecasts and a better model fit. It would also be worthwhile comparing these findings across countries and over time. In addition, future work should also include model forms that include such things as structural breaks, non-linearities, regime switching, and time varying coefficient estimation to the forecasts in light of the observations presented above.

6.0 References

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