ACTA SCIENTIAE
ISSN: 2178-7727
DOI: 10.17648/acta.scientiae.6369

CLOSED SETS AND CONTINUOUS FUNCTIONS VIA FUZZY QUADRIGEMINAL TOPOLOGICAL SPACES

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Abstract

Fuzzy Quadrigeminal sets is a new approach introduced for the purpose of measuring degree of belongingness of elements to a subset. Fuzzy Quadrigeminal set helps us to draw some conclusion whether a positive result be possible or not in an uncertain situation. The aim of this paper is to elicit some basic properties concerned to closed sets and continuous functions via Fuzzy Quadrigeminal Topological spaces which was proposed by Razzag et al in "Picture fuzzy topological spaces and associated continuous functions" [13]. Also, it tries to exhibit some properties of generalized closed sets. Existence of open cover for a Fuzzy Quadrigeminal Topological Space is proved, and hence, the compactness of spaces is derived. The concept of relations between two Fuzzy Quadrigeminal sets is developed. Thus, this article explores some topological concepts via Fuzzy Quadrigeminal Topological spaces.

Keywords: Fuzzy Quadrigeminal Set, Fuzzy Quadrigeminal Topological spaces, Open Fuzzy Quadrigeminal set, Closed Fuzzy Quadrigeminal set, Continuous Functions.

1. **Introduction**

The new notion of assigning degree values, namely degree of membership, to elements of a crisp set was first proposed by A. Zadeh [1] in the year 1965. Zadeh's single value assignment to elements later modified by K. Atanassov [5] in 1983. K. Atanassov assigned two degree values, namely degree of membership and degree of non-membership, to an element of a set. Later, in 1995, F. Smarandache [6] extended the concept of assigning degree values to an element of a set by giving three degree values. Smarandache added 'the degree of indeterminacy', a degree value to address lack of knowledge about an element. The notion of Fuzzy Quadrigeminal Sets was first proposed by [14]. Fuzzy Quadrigeminal Sets came up with an idea of assigning four degree values to an element on the basis of its belongingness to the set. The four membership degrees are assigned on the basis of 1. Extreme-belongingness 2. Very-belongingness 3. Moderate-belongingness 4. Weak-belongingness. Also, the authors exhibited that Fuzzy Quadrigeminal sets form a topology and brought forth Fuzzy Quadrigeminal Topological Space. Open sets, closed sets and interior of a set are defined. In this paper closed sets and continuous function via Fuzzy Quadrigeminal Topological Space are studied and some of their properties are explained.

2. **Preliminaries**

Some of the basic notions and definitions are presented in this section and then the notions of closed sets and continuous function are introduced.

Definition 2.1

Let Ƒ be a non-empty set of objects. A Fuzzy Quadrigeminal set [14] [FQs in short] Ғ in Ƒ is of the form

$$\text{Ғ} = \{ < a, \Upsilon_{\text{Ғ}}(a), \Omega_{\text{Ғ}}(a), \mho_{\text{Ғ}}(a), \partial_{\text{Ғ}}(a) > \ : \ a \in \text{Ƒ} \}$$

where $\Upsilon_{\text{Ғ}} : \text{Ƒ} \rightarrow [0,1]$ is called the degree of extreme - belongingness to Ғ

$\Omega_{F} : Ғ \rightarrow [0,1]$ is called the degree of very - belongingness to Ғ
$\mho_{F} : Ғ \rightarrow [0,1]$ is called the degree of moderate – belongingness to Ғ
$\partial_{F} : Ғ \rightarrow [0,1]$ is called the degree of weak- belongingness to Ғ
such that $\Upsilon_{F}(a)+\Omega_{F}(a)+\mho_{F}(a)+\partial_{F}(a)\leq 1$.

Example 2.1
Let Ғ = { x ,y } be a universal set. A FQ set Ғ is of the form
Ғ = { < x, 0.75,0.15,0.06,0.02 > , < y, 0.02,0.1,0.7,0.1 > }

Let Ғ denotes the family of all FQ sets over Ғ .
Definitions 2.2
Empty set
The empty FQs [14] may be defined as follows.
$0_{Ғ}$ = { < x, 0,0,0,1 >}
$0_{Ғ}$ = { < x, 0,0,1,0 >}
$0_{Ғ}$ = { < x, 0,1,0,0 >}
$0_{Ғ}$ = { < x, 0,0,0,0> }

Universal Set
In the same way, the set 1_{F} [14] may be defined as
$1_{Ғ}$ = { < x, 1,0,0,0 > }

3. Algebraic Properties
The set operations over FQ sets were defined in [14]. In this section we define some algebraic operations over the FQ sets of the universal set Q and present some results.
Definition 3.1: Algebraic Sum
The algebraic sum of two FQ sets S and T is defined as
$S \oplus T = \{ < q, \Upsilon_S(q) + \Upsilon_T(q) - \Upsilon_S(q)\,\Upsilon_T(q),\ \Omega_S(q)\,\Omega_T(q),$
$\mho_S(q)\,\mho_T(q),\ \partial_S(q)\ \partial_T(q) > \}$
Definition 3.2: Algebraic Product
The algebraic product of two FQ sets S and T is defined as
$S \otimes T = \{ < q, \Upsilon_S(q)\,\Upsilon_T(q),\ \Omega_S(q)\,\Omega_T(q),\ \mho_S(q)\,\mho_T(q),\ \partial_S(q) + \partial_T(q) -$
$\partial_S(q)\ \partial_T(q) > \}$
Example 3.1: Let S = {< q_1, 0.2, 0.6, 0.1, 0.05 >, < q_2, 0.7, 0.15, 0.09, 0.03 >}
Let T = {< q_1, 0.8, 0.1, 0.07, 0.02 >, < q_2, 0.08, 0.1, 0.2, 0.6 >}
Then, $S \oplus T = \{ < q_1, 0.84,\ 0.06,\ 0.007, 0.001 > , < q_2, 0.724, 0.015, 0.018, 0.018 >\}$
$S \otimes T = \{ < q_1, 0.16,\ 0.06,\ 0.007,\ 0.069 >,\ < q_2, 0.056,\ 0.015,\ 0.018,\ 0.612 >\}$
Definition 3.3: The scalar multiplication operator over a FQ set P, denoted by nP, is defined as
$nP = \{ < q, 1 - [\,1 - \Upsilon_S(q)]^n, \Omega_S(q)^n, \mho_S(q)^n, \partial_S(q)^n > | q \in Q \}$
Definition 3.4: The exponentiation operator, denoted by S^n , is defined as
$S^n = \{ < q, [\,\Upsilon_S(q)]^n, \Omega_S(q)^n, \mho_S(q)^n, 1 - [\,1 - \partial_S(q)]^n > \}$
Theorem: For every S , T ∈ Ғ, we have $S \otimes T \subseteq S \oplus T$
Proof: $S \oplus T = \{ < q, \Upsilon_S(q) + \Upsilon_T(q) - \Upsilon_S(q)\,\Upsilon_T(q),\ \Omega_S(q)\,\Omega_T(q),$
$\mho_S(q)\,\mho_T(q),\ \partial_S(q)\ \partial_T(q) > \}$
$S \otimes T = \{ < q, \Upsilon_S(q)\,\Upsilon_T(q),\ \Omega_S(q)\,\Omega_T(q),\ \mho_S(q)\,\mho_T(q),\ \partial_S(q) + \partial_T(q) -$
$\partial_S(q)\ \partial_T(q) > \}$
$$\Upsilon_S(q)\,\Upsilon_T(q) \leq \Upsilon_S(q) + \Upsilon_T(q) - \Upsilon_S(q)\,\Upsilon_T(q)$$

$$\Omega_S(q)\,\Omega_T(q) = \Omega_S(q)\,\Omega_T(q)$$
$$\mho_S(q)\,\mho_T(q) = \mho_S(q)\,\mho_T(q)$$
$$\partial_S(q) + \partial_T(q) - \partial_S(q)\,\partial_T(q) \geq \partial_S(q)\,\partial_T(q)$$

Clearly, $S \otimes T \subseteq S \oplus T$

Results:

1. $S \oplus S \supseteq S$
2. $S \otimes S \subseteq S$
3. $S \oplus (S \otimes T) \supseteq S$ and $S \otimes (S \oplus T) \subseteq S$
4. $P \oplus (S \cup T) = (P \oplus S) \cup (P \oplus T)$
5. $P \oplus (S \cap T) = (P \oplus S) \cap (P \oplus T)$
6. $P \otimes (S \cup T) = (P \otimes S) \cup (P \otimes T)$
7. $P \otimes (S \cap T) = (P \otimes S) \cap (P \otimes T)$
8. $(S \cup T) \oplus (S \cap T) = S \oplus T$
9. $(S \cup T) \otimes (S \cap T) = S \otimes T$
10. $(S \oplus T) \cap (S \otimes T) = (S \otimes T)$
11. $(S \oplus T) \cup (S \otimes T) = (S \oplus T)$

Theorem: For any S, $T \in$ Ƒ, we have the following hold

(i) $(S \oplus T)^c = S^c \otimes T^c$

(ii) $(S \otimes T)^c = S^c \oplus T^c$

(iii) $(S \oplus T)^c \subseteq S^c \oplus T^c$

(iv) $(S \otimes T)^c \supseteq S^c \otimes T^c$

Proof: (i) and (ii) are straight forward. We prove the (iii) and (iv).

(iii) $S \oplus T = \{ < q, \Upsilon_S(q) + \Upsilon_T(q) - \Upsilon_S(q)\,\Upsilon_T(q),\ \Omega_S(q)\,\Omega_T(q),\ \mho_S(q)\,\mho_T(q),\ \partial_S(q)\,\partial_T(q) > \}$

$(S \oplus T)^c = \{ < q, \partial_S(q)\,\partial_T(q), \mho_S(q)\,\mho_T(q), \Omega_S(q)\,\Omega_T(q), \Upsilon_S(q) + \Upsilon_T(q) - \Upsilon_S(q)\,\Upsilon_T(q) > \}$

$S^c = \{ < q, \partial_S(q), \mho_S(q), \Omega_S(q), \Upsilon_S(q) > \}$ and

$T^c = \{ < q, \partial_T(q), \mho_T(q), \Omega_T(q), \Upsilon_T(q) > \}$

$$S^c \oplus T^c = \{ < q, \partial_S(q) + \partial_T(q) - \partial_S(q)\,\partial_T(q), \mho_S(q)\,\mho_T(q), \Omega_S(q)\,\Omega_T(q), \Upsilon_S(q)\,\Upsilon_T(q) > \}$$

We have $\partial_S(q)\,\partial_T(q) \leq \partial_S(q) + \partial_T(q) - \partial_S(q)\,\partial_T(q)$

$$\Upsilon_S(q) + \Upsilon_T(q) - \Upsilon_S(q)\,\Upsilon_T(q) \geq \Upsilon_S(q)\,\Upsilon_T(q)$$

Thus, $(S \oplus T)^c \subseteq S^c \oplus T^c$

(iv) $S \otimes T = \{ < q, \Upsilon_S(q)\,\Upsilon_T(q),\ \Omega_S(q)\,\Omega_T(q),\ \mho_S(q)\,\mho_T(q),\ \partial_S(q) + \partial_T(q) - \partial_S(q)\,\partial_T(q) > \}$

$$(S \otimes T)^c = \{ < q, \partial_S(q) + \partial_T(q) - \partial_S(q)\,\partial_T(q), \mho_S(q)\,\mho_T(q), \Omega_S(q)\,\Omega_T(q), \Upsilon_S(q)\,\Upsilon_T(q) > \}$$

$S^c \otimes T^c = \{ < q, \partial_S(q)\,\partial_T(q), \mho_S(q)\,\mho_T(q), \Omega_S(q)\,\Omega_T(q), \Upsilon_S(q) + \Upsilon_T(q) - \Upsilon_S(q)\,\Upsilon_T(q) > \}$

Clearly, $(S \otimes T)^c \supseteq S^c \otimes T^c$

Theorem: For $n, n_1, n_2 > 0$ and for S,T ∈ Ƒ, we have

(i) $n(S \oplus T) = nS \oplus nT$

(ii) $n_1 S \oplus n_2 S = (n_1 + n_2) S$

(iii) $(S \otimes T)^n = S^n \otimes T^n$

(iv) $S^{n_1} \otimes S^{n_2} = S^{n_1 + n_2}$

Proof: It is easy.

4. FUZZY QUADRIGEMINAL TOPOLOGICAL SPACES[14]

Definition 4.1

Let Γ be a collection of sets from Ƒ. Then Γ is called a topology on Ƒ if

(i) $0_Ƒ$, $1_Ƒ$ ϵ Γ

(ii) $Ƒ_1 \cap Ƒ_2$ ϵ Γ , whenever $Ƒ_1$, $Ƒ_2$ ϵ Γ

(iii) $\bigcup_{i\in j} Ƒ_i \in$ Γ, if $Ƒ_i \in$ Γ

The pair (Q,Γ) is called Fuzzy Quadrigeminal Topological space. In short we use FQTS for Fuzzy Quadrigeminal Topological Space .

Definition 4.2

The members of Γ are called fuzzy quadrigeminal open sets (FQOS in short) in Ƒ .
If c($Ƒ_1$) ϵ Γ, then $Ƒ_1$ ϵ Ƒ is said to be fuzzy quadrigeminal closed set (FQCS in short) in Ƒ.

Example 4.1

Let Q = { q_1 , q_2 } be a universe of discourse.
Let Q_1 , Q_2 ,Q_3 and Q_4 be FQ sets in X such that

Q_1 = { < q_1 , 0.2, 0.6, 0.1, 0.05 > , < q_2 , 0.7, 0.15, 0.09, 0.03 > }
Q_2 = { < q_1 , 0.05, 0.1, 0.7, 0.15 > , < q_2 , 0.08, 0.1, 0.2, 0.6 > }
Q_3 = { < q_1 , 0.2, 0.1, 0.1, 0.05 > , < q_2 , 0.7, 0.1, 0.09, 0.03 > }
Q_4 = { < q_1 , 0.05, 0.1, 0.1, 0.15> , < q_2 , 0.08, 0.1, 0.09, 0.6 > }

Ꞇ = { 0_Q , 1_Q , Q_1 , Q_2 ,Q_3 , Q_4 } forms a topology on Q and the pair (Q , Ꞇ) is a FQTS.

Definition 4.3

Let { Q, Ꞇ } be a FQTS over Q and Q_1 ϵ Ƒ. Then, the FQ interior of Q_1 , denoted by FQ$\mathscr{I}$(Q_1), is the union of all FQ open subsets of Q_1.

Remark 4.1

1. Q_1 is FQOS if and only if Q_1 = FQ$\mathscr{I}$(Q_1)

5. Fuzzy Quadrigeminal Closed Set

In a topological space open sets and closed sets are very important since they help to distinguish points and subsets of a space. Also they help us to define continuous functions between spaces. This section is dedicated to closed sets.

Definition 5.1 Let { Q, Ꞇ } be a FQTS over Q and Q_1 ϵ Ƒ. Then, the FQ closure of Q_1 denoted by FQ$\mathcal{C}$(Q_1) , is defined by

FQ$\mathcal{C}$(Q_1)= $\cap\{ P : P \text{ is a } FQCS \text{ in } Q \text{ and } Q_1 \subseteq P \}$.

Remarks 5.1

1. Q_1 is FQCS if and only if Q_1 = FQ$\mathcal{C}$(Q_1)
2. If P and S are FQCS then $P \cup S$ is a FQCS.
3. The intersection of two FQCS is a FQCS.

6. FQ-Continuous Maps

Definition 6.1: Let $h : X \to Y$ be a map and P be a FQ subset of X. The four measures of belongingness for image of P, denoted by h[P] are determined by

$$\Upsilon_{h[P]}(y) = \begin{cases} \sup_{z\in h^{-1}(y)} \Upsilon_P(z), & if\ h^{-1}(y) \neq \emptyset \\ 0\ , otherwise \end{cases}$$

$$\Omega_{h[P]}(y) = \begin{cases} \inf_{z\in h^{-1}(y)} \Omega_P(z), & if\ h^{-1}(y) \neq \emptyset \\ 0\ , otherwise \end{cases}$$

$$\mho_{h[P]}(y) = \begin{cases} \inf_{z\in h^{-1}(y)} \mho_P(z), & if\ h^{-1}(y) \neq \emptyset \\ 0\ , otherwise \end{cases}$$

$$\partial_{h[P]}(y) = \begin{cases} \inf_{z\in h^{-1}(y)} \partial_P(z), & if\ h^{-1}(y) \neq \emptyset \\ 1\ , otherwise \end{cases}$$

Let $h : X \to Y$ be a map and Q be a FQ subset of Y. Then, the four measures of belongingness of preimage of Q, denoted by $h^{-1}[Q]$, are calculated by

$$\Upsilon_{h^{-1}[Q]}(x) = \Upsilon_Q(h(x))$$
$$\Omega_{h^{-1}[Q]}(x) = \Omega_Q(h(x))$$
$$\mho_{h^{-1}[Q]}(x) = \mho_Q(h(x))$$
$$\partial_{h^{-1}[Q]}(x) = \partial_Q(h(x))$$

Theorem 6.1: Let $h : U \to V$ be a map and P and Q are FQ subsets of U and V, respectively. Then, the following are true.

(i) $h^{-1}[Q^c] = h^{-1}[Q]^c$
(ii) $h[P]^c \subseteq h[P^c]$
(iii) For any two FQ subsets Q_1 and Q_2, $h^{-1}[Q_1] \subseteq h^{-1}[Q_2]$ whenever $Q_1 \subseteq Q_2$ in V
(iv) For any two FQ subsets P_1 and P_2, we have $h[P_1] \subseteq h[P_2]$ whenever $P_1 \subseteq P_2$ in U
(v) $h[h^{-1}[Q]] \subseteq Q$
(vi) $P \subseteq h^{-1}[h[P]]$

Proof: It is clear.

Definition 6.2 : Let P and U be two FQ subsets in a FQTS (Q, Ꞇ). We say U is a neighborhood of P , in short nbd, if there is an FQOS W such that $P \subseteq W \subseteq U$.

Theorem 6.2: A FQ subset P is open if and only if it contains a nbd of its each subset.

Proof: It is clear.

Definition 6.3 : A map h : (X, $Ꞇ_1$) → (Y, $Ꞇ_2$) is said to be FQ-continuous if for any FQ subset P of X and for any nbd V of h[P] there is a nbd U of P such that $h[U] \subseteq V$.

Theorem 6.3: The following statements are equivalent for a map h: (X, $Ꞇ_1$) → (Y, $Ꞇ_2$)

(a) h is FQ-continuous
(b) For each FQ set P of X and each nbd V of h[P], there is a nbd U of P such that for each $Q \subseteq U$, we have $h[Q] \subseteq V$.

(c) For each FQ set P of X and for each nbd V of h[P], there is a nbd U of P such that U $\subseteq h^{-1}[V]$.
(d) For each FQ set P of X and for each nbd V of h[P], $h^{-1}[V]$ is a nbd of P.

Proof:
(a) => (b) Let h be a FQ-continuous map and P is a FQ set of X. Let V be a nbd of h[P]. Since h is FQ-continuous, for any nbd U of P, h[U] ⊆ V.
so, if Q ⊆ P, h[Q] ⊆ h[P] ⊆ V.
(b) => (c) From (b) for each Q ⊆ U, we have h[Q] ⊆ V

$$=> Q \subseteq f^{-1}f[Q] \subseteq f^{-1}[V]$$

Since Q is arbitrary, U $\subseteq f^{-1}[V]$.
(c) => (d) Since U is a nbd of P, there exists an open set W such that P ⊆ W ⊆ U
So, from (c), P ⊆ W ⊆ U $\subseteq h^{-1}[V]$
i.e, P ⊆ W $\subseteq h^{-1}[V]$ and hence $h^{-1}[V]$ is a nbd.
(d) => (a) Since, $h^{-1}[V]$ is a nbd, there is a FQOS W such that,

$$P \subseteq W \subseteq h^{-1}[V].$$

$$h[W] \subseteq h\left[h^{-1}[V]\right] \subseteq V$$

As W is a FQOS, it is a nbd of P. Thus, h is FQ-continuous since h[W] $\subseteq V$.
Definition 6.4: A map h : (X, $Ꞇ_1$) → (Y, $Ꞇ_2$) is said to be FQ-continuous if and only if the preimage of each FQOS in Y is a FQOS in X.
Theorem 6.4: The FQ-continuity of h: (X, $Ꞇ_1$) → (Y, $Ꞇ_2$) implies that h: X → Y is injective.
Proof: Suppose the function h : X → Y is non-injective. Then, for any two x_1, x_2 we have $h(x_1) = h(x_2)$ for $x_1 \neq x_2$. Consider a FQ set P of X such that
(i) $\Upsilon_P(x_i) = 1$ and $\Omega_P(x_i) = 0$ for all $x_i \in X - \{x_1\}$
(ii) $Ʊ_P(x_j) = \partial_P(x_j) = 0$ for all $x_j \in X$
(iii) $\Upsilon_P(x_1) + \Omega_P(x_1) = 1$, and $\Upsilon_P(x_i)$, $\Omega_P(x_i) \in (0,1)$.
(iv) $\Omega_P(x_1)$ is greater than every R ∈ $Ꞇ_1$ such that $\Omega_R(x_1) < 1$.
Then P is not contained in any R ∈ $Ꞇ_1$ for which $\Omega_R(x_1) < 1$.
If $R^/ \in Ꞇ_1$ such that $\Omega_{R^/}(x_1) = 1$, then $\Upsilon_{R^/}(x_1) = 0$ and $\Upsilon_P(x_1) > 0$. So, P is not contained in any $R^/ \in Ꞇ_1$ such that $\Omega_{R^/}(x_1) = 1$. Further, 1_Y is a nbd of h[P]. But, there is no nbd for P. This contradicts the fact that h is continuous. Thus, h is an injective function.
Definition 6.5 : A family { Q_i : i ∈ I } of FQOS of X is said to be a FQ open cover of X if ∪ { Q_i : i ∈ I } = 1_X .
Definition 6.6 : A FQTS (X, $Ꞇ_1$) is called FQ compact if each FQ open cover of X has a finite subcover.
Definition 6.7 : A family { Q_i : i ∈ I } of FQ sets has the finite intersection property iff the intersection of the members of each finite subfamily of { Q_i : i ∈ I } is not equal to 0_X .
Results
1. A FQTS is compact if and only if each family of FQCS which has the finite intersection property has a nonempty intersection.
2. Let h be a FQ- continuous function carrying the FQ compact space X onto the FQTS Y. Then, Y is compact.

7. **Generalized Closed Sets And FQg-continuous Functions**
The notion of generalized closed sets and g-continuous Functions are defined in this section.

Definition 7.1 : A FQ set A of a FQTS (X, Ꞇ) is called a FQ generalized closed (FQg-closed) if FQC̃(A) ⊆ O whenever A ⊆ O and O is a FQOS.
Definition 7.2 : The complement of a FQg-closed set is called FQg-open set.
Remark 7.1 : Every FQCS (FQOS) is FQg-closed (FQg-open) set, but the converse need not be true.
Definition 7.3 : A mapping h : (X, $Ꞇ_1$) → (Y, $Ꞇ_2$) is said to be FQg-continuous if the inverse image of every FQCS of Y is FQg-closed in X.
Definition 7.4: Let (X, Ꞇ) be a FQTS. The generalized closure of a FQ set P of X, denoted by gC̃(P), is the intersection of all FQg-closed sets of X which contains P.
Remark: P ⊆ gC̃(P) ⊆ FQC̃(P) for any FQ set P of X.
Theorem 7.1: If a mapping h : (X, $Ꞇ_1$) → (Y, $Ꞇ_2$) is FQg-continuous, then
h(gC̃(P)) ⊆ FQC̃(h(P))
Proof: Let P be a FQ set of X. Then FQC̃(h(P)) is a FQ-closed set of Y. Since h is FQg-continuous, h^{-1}(FQC̃(h(P)) is FQg-closed in X.
But, we have P ⊆ h^{-1}(FQC̃(h(P))) and so gC̃(P) ⊆ gC̃ (h^{-1}(FQC̃(h(P)))).
Hence, h(gC̃(P)) ⊆ FQC̃(h(P)).

8. FUZZY QUADRIGEMINAL RELATIONS

Let E,F and G be non-empty sets of discourse.
Definition 8.1: A FQ-relation, denoted by Ř, is a collection from the set E × F given by
Ř = { < (e , f) , $\Upsilon_{Ř}$ (e , f), $\Omega_{Ř}$(e , f), $Ʊ_{Ř}$(e , f), $\partial_{Ř}$(e , f) > | e ∈ E , f ∈ F }
Where $\Upsilon_{Ř}$: E × F → [0,1] , $\Omega_{Ř}$: E × F → [0,1] , $Ʊ_{Ř}$: E × F → [0,1] , $\partial_{Ř}$: E × F → [0,1]
and also satisfying the condition 0 ≤ $\Upsilon_{Ř}$ (e , f)+ $\Omega_{Ř}$(e , f) + $Ʊ_{Ř}$(e , f) + $\partial_{Ř}$(e , f) ≤1
for every (e , f) ∈ E × F.
FQ- Ř(E × F) denotes the set of all FQ-relation in E × F
Definition 8.2: Let Ř ∈ FQ- Ř(E × F). The inverse relation $Ř^{-1}$ between F and E is defined as

$\Upsilon_{Ř}^{-1}$ (f , e) = $\Upsilon_{Ř}$ (e , f), $\Omega_{Ř}^{-1}$(f , e) = $\Omega_{Ř}$(e , f),
$Ʊ_{Ř}^{-1}$(f , e) = $Ʊ_{Ř}$(e , f) , $\partial_{Ř}^{-1}$(f , e) = $\partial_{Ř}$(e , f) for all (e , f) ∈ E × F.

We state here some properties of FQ-relation.
Definition 8.3: Let Ř and Š be two FQ-relations between E and F. For every (e , f) ∈ E × F
We define,

(a) Ř ≤ Š ⟺ $\Upsilon_{Ř}$ (e , f)≤ $\Upsilon_{Š}$ (e , f), $\Omega_{Ř}$(e , f) ≤ $\Omega_{Š}$(e , f) ,$Ʊ_{Ř}$(e , f) ≤ $Ʊ_{Š}$(e , f) $\partial_{Ř}$(e , f) ≥ $\partial_{Š}$(e , f)
(b) Ř ⋁ Š = { < (e , f), $\Upsilon_{Ř}$ (e , f) ∨ $\Upsilon_{Š}$ (e , f), $\Omega_{Ř}$(e , f) ∧ $\Omega_{Š}$(e , f) ,$Ʊ_{Ř}$(e , f) ∧ $Ʊ_{Š}$(e , f) , $\partial_{Ř}$(e , f) ∧ $\partial_{Š}$(e , f) > }
(c) Ř ∧ Š = { < (e , f), $\Upsilon_{Ř}$ (e , f) ∧ $\Upsilon_{Š}$ (e , f), $\Omega_{Ř}$(e , f) ∧ $\Omega_{Š}$(e , f) ,$Ʊ_{Ř}$(e , f) ∧ $Ʊ_{Š}$(e , f) , $\partial_{Ř}$(e , f) ∨ $\partial_{Š}$(e , f) > }
(d) c(Ř) = { < (e,f), $\partial_{Ř}$(e , f), $Ʊ_{Ř}$(e , f), $\Omega_{Ř}$(e , f), $\Upsilon_{Ř}$ (e , f) > | e ∈ E , f ∈ F}

Results: Let Ř ,Š and Ť be FQ-relations.

1. $(Ř^{-1})^{-1} = Ř$
2. $Ř \leq Š \Longrightarrow Ř^{-1} \leq Š^{-1}$
3. $(Ř \vee Š)^{-1} = Ř^{-1} \vee Š^{-1}$
4. $(Ř \wedge Š)^{-1} = Ř^{-1} \wedge Š^{-1}$
5. $Ř \wedge (Š \vee Ť) = (Ř \wedge Š) \vee (Ř \wedge Ť)$
6. $Ř \vee (Š \wedge Ť) = (Ř \vee Š) \wedge (Ř \vee Ť)$
7. $Ř \wedge Š \leq Ř,\ Ř \wedge Š \leq Š$

8. If $(\check{R} \geq \check{S})$ *and* $(\check{R} \geq \check{T})$ *then* $\check{R} \geq \check{S} \vee \check{T}$
9. If $(\check{R} \leq \check{S})$ *and* $(\check{R} \leq \check{T})$ *then* $\check{R} \leq \check{S} \wedge \check{T}$

Conclusion

In this paper we exhibited some algebraic properties of fuzzy quadrigeminal sets. Also, we presented important concepts like closed sets, neighborhoods, continuous functions and generalized closed sets via Fuzzy Quadrigeminal Topological Spaces.

Conflict of Interest:

On behalf of all authors, the corresponding author states that there is no conflict of interest.

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